Act. #32

Arithmetic Sequences

Arithmetic Means The **arithmetic means** of an arithmetic sequence are the terms between any two nonsuccessive terms of the sequence.

To find the k arithmetic means between two terms of a sequence, use the following steps.

Step 1	Let the two terms given be a_1 and a_n, where $n = k + 2$.
Step 2	Substitute in the formula $a_n = a_1 + (n - 1)d$.
Step 3	Solve for d, and use that value to find the k arithmetic means: $a_1 + d, a_1 + 2d, \ldots, a_1 + kd$.

Example **Find the five arithmetic means between 37 and 121.**

You can use the nth term formula to find the common difference. In the sequence, 37, ?, ?, ?, ?, ?, 121, ..., a_1 is 37 and a_7 is 121.

$a_n = a_1 + (n - 1)d$	Formula for the nth term
$121 = 37 + (7 - 1)d$	$a_1 = 37$, $a_7 = 121$, $n = 7$
$121 = 37 + 6d$	Simplify.
$84 = 6d$	Subtract 37 from each side.
$d = 14$	Divide each side by 6.

Now use the value of d to find the five arithmetic means.

$$37 \overset{+14}{\rightarrow} 51 \overset{+14}{\rightarrow} 65 \overset{+14}{\rightarrow} 79 \overset{+14}{\rightarrow} 93 \overset{+14}{\rightarrow} 107 \overset{+14}{\rightarrow} 121$$

The arithmetic means are 51, 65, 79, 93, and 107.

Exercises

Find the arithmetic means in each sequence.

1. 5, ?, ?, ?, −3
2. 18, ?, ?, ?, −2
3. 16, ?, ?, 37
4. 108, ?, ?, ?, ?, 48
5. −14, ?, ?, ?, −30
6. 29, ?, ?, ?, 89
7. 61, ?, ?, ?, ?, 116
8. 45, ?, ?, ?, ?, ?, 81
9. −18, ?, ?, ?, 14
10. −40, ?, ?, ?, ?, ?, −82
11. 100, ?, ?, 235
12. 80, ?, ?, ?, ?, −30
13. 450, ?, ?, ?, 570
14. 27, ?, ?, ?, ?, ?, 57
15. 125, ?, ?, ?, 185
16. 230, ?, ?, ?, ?, ?, 128
17. −20, ?, ?, ?, ?, 370
18. 48, ?, ?, ?, 100