Act. #93

Arithmetic Series

Sigma Notation A shorthand notation for representing a series makes use of the Greek letter Σ. The **sigma notation** for the series $6 + 12 + 18 + 24 + 30$ is $\sum_{n=1}^{5} 6n$.

Example **Evaluate $\sum_{k=1}^{18} (3k + 4)$.**

The sum is an arithmetic series with common difference 3. Substituting $k = 1$ and $k = 18$ into the expression $3k + 4$ gives $a_1 = 3(1) + 4 = 7$ and $a_{18} = 3(18) + 4 = 58$. There are 18 terms in the series, so $n = 18$. Use the formula for the sum of an arithmetic series.

$S_n = \frac{n}{2}(a_1 + a_n)$ Sum formula

$S_{18} = \frac{18}{2}(7 + 58)$ $n = 18, a_1 = 7, a_n = 58$

$= 9(65)$ Simplify.

$= 585$ Multiply.

So $\sum_{k=1}^{18} (3k + 4) = 585$.

Subtract two numbers then add one.

Exercises Use: $\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$

Find the sum of each arithmetic series.

1. $\sum_{n=1}^{20} (2n + 1)$
2. $\sum_{n=5}^{25} (x - 1)$
3. $\sum_{k=1}^{18} (2k - 7)$
4. $\sum_{r=10}^{75} (2r - 200)$
5. $\sum_{x=1}^{15} (6x + 3)$
6. $\sum_{t=1}^{50} (500 - 6t)$
7. $\sum_{k=1}^{80} (100 - k)$
8. $\sum_{n=20}^{85} (n - 100)$
9. $\sum_{s=1}^{200} 3s$
10. $\sum_{m=14}^{28} (2m - 50)$
11. $\sum_{p=1}^{36} (5p - 20)$
12. $\sum_{j=12}^{32} (25 - 2j)$
13. $\sum_{n=18}^{42} (4n - 9)$
14. $\sum_{n=20}^{50} (3n + 4)$
15. $\sum_{j=5}^{44} (7j - 3)$